The **testcorr** Add-in

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Abstract

The EViews add-in **testcorr** implements standard and robust procedures for testing the significance of the autocorrelation in univariate data and the cross-correlation in bivariate data. It also includes tests for the significance of pairwise Pearson correlation in multivariate data and the i.i.d. property for univariate data. The standard testing procedures on significance of correlation are used commonly by practitioners while their robust versions were developed in Dalla *et al.* (2019), where the tests for i.i.d. property can be also found. This document briefly outlines the testing procedures and provides simple examples.

Keywords: autocorrelation, cross-correlation, Pearson correlation, i.i.d.

1 Introduction

Inference on the significance of the autocorrelation $\rho_k = \text{corr}(x_t, x_{t-k})$ or the cross-correlation $\rho_{xy,k} = \text{corr}(x_t, y_{t-k})$ is a common first step in the analysis of univariate $\{x_t\}$ or bivariate $\{x_t, y_t\}$ time series data. Moreover, it is common to test the significance of pair-wise correlations $\rho_{x_i x_j} = \text{corr}(x_{it}, x_{jt})$ in multivariate $\{x_{1t}, x_{2t}, ..., x_{pt}\}$ data, cross-sectional or time series. Standard inference procedures[1] are valid for i.i.d. univariate or mutually independent bivariate/multivariate data and their size can be significantly distorted otherwise, in particular, by heteroscedasticity and dependence. The robust methods given in Dalla *et al.* (2019) allow testing for significant autocorrelation/cross-correlation/correlation under more general settings, e.g., they allow for heteroscedasticity and dependence in each series and mutual dependence across series.

The add-in **testcorr** implements the standard and robust procedures for testing significance of autocorrelation and cross-correlation, respectively. Moreover, the add-in evaluates the sample Pearson correlation matrix for multivariate data with robust p-values for testing significance of its elements. The add-in also conducts testing procedures for the i.i.d. property of univariate data introduced in Dalla *et al.* (2019). Sections 2-5 describe the testing procedures and provide examples. Section 6 outlines some suggestions relating to the application of the testing procedures.

[1]Like those implemented by the **Correlogram**, the **Cross-correlogram** and the **Covariance Analysis**.

2 Testing zero autocorrelation

For a univariate time series $\{x_t\}$, given a sample $x_1, ..., x_n$, the null hypothesis $H_0 : \rho_k = 0$ of no autocorrelation at lag $k = 1, 2, ...$ is tested at α significance level using the sample autocorrelation $\widehat{\rho}_k$ and the $100(1-\alpha)\%$ confidence band (CB) for zero autocorrelation, obtained using the corresponding t-type statistics (t_k "standard" and $\widetilde{t}_k$ "robust").[2] The null hypothesis $H_0 : \rho_1 = ... = \rho_m = 0$ of no autocorrelation at cumulative lags $m = 1, 2, ...$ is tested using portmanteau type statistics (Ljung-Box LB_m "standard" and $\widetilde{Q}_m$ "robust"). The following notation is used.

Standard procedures:

$$CB(100(1-\alpha)\%) = (-z_{\alpha/2}/\sqrt{n}, z_{\alpha/2}/\sqrt{n}), \quad t_k = \sqrt{n}\widehat{\rho}_k, \quad LB_m = (n+2)n \sum_{k=1}^{m} \frac{\widehat{\rho}_k^2}{n-k}.$$

Robust procedures:

$$CB(100(1-\alpha)\%) = (-z_{\alpha/2}\frac{\widehat{\rho}_k}{\widetilde{t}_k}, z_{\alpha/2}\frac{\widehat{\rho}_k}{\widetilde{t}_k}), \quad \widetilde{t}_k = \frac{\sum_{t=k+1}^{n} e_{tk}}{\left(\sum_{t=k+1}^{n} e_{tk}^2\right)^{1/2}}, \quad \widetilde{Q}_m = \widetilde{t}' \, \widehat{R}^{*\,-1} \widetilde{t},$$

where $e_{tk} = (x_t - \bar{x})(x_{t-k} - \bar{x})$, $\bar{x} = n^{-1} \sum_{t=1}^{n} x_t$, $\widetilde{t} = (\widetilde{t}_1, ..., \widetilde{t}_m)'$ and $\widehat{R}^* = (\widehat{r}_{jk}^*)$ is a matrix with elements $\widehat{r}_{jk}^* = \widehat{r}_{jk} I(|\tau_{jk}| > \lambda)$ where λ is the threshold,

$$\widehat{r}_{jk} = \frac{\sum_{t=\max(j,k)+1}^{n} e_{tj} e_{tk}}{(\sum_{t=\max(j,k)+1}^{n} e_{tj}^2)^{1/2} (\sum_{t=\max(j,k)+1}^{n} e_{tk}^2)^{1/2}}, \quad \tau_{jk} = \frac{\sum_{t=\max(j,k)+1}^{n} e_{tj} e_{tk}}{(\sum_{t=\max(j,k)+1}^{n} e_{tj}^2 e_{tk}^2)^{1/2}}.$$

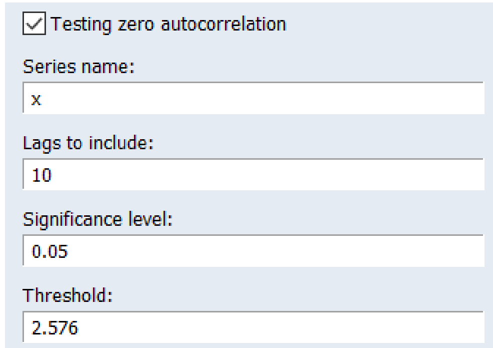
Applying standard and robust tests, at significance level α, $H_0 : \rho_k = 0$ is rejected when $\widehat{\rho}_k \notin CB(100(1-\alpha)\%)$ or $|t_k|, |\widetilde{t}_k| > z_{\alpha/2}$. In turn, $H_0 : \rho_1 = ... = \rho_m = 0$ is rejected when $LB_m, \widetilde{Q}_m > \chi^2_{m,\alpha}$. Here, $z_{\alpha/2}$ and $\chi^2_{m,\alpha}$ stand for the upper $\alpha/2$ and α quantiles of $N(0,1)$ and χ^2_m distributions.

Example

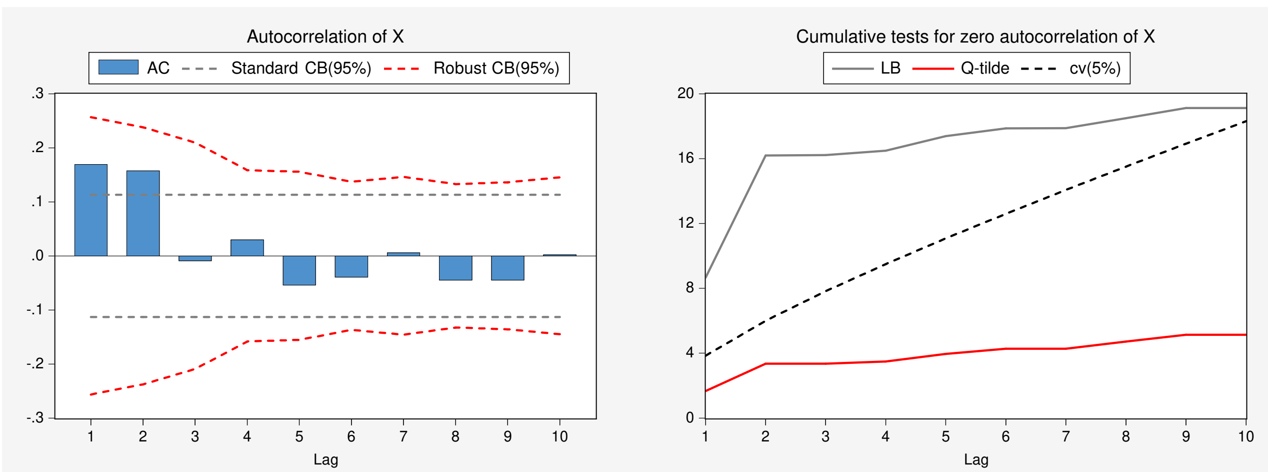
We provide an example to illustrate testing for zero autocorrelation of a univariate time series $\{x_t\}$. We simulate $n = 300$ data as GARCH(1,1): $x_t = \sigma_t \varepsilon_t$ with $\sigma_t^2 = 1 + 0.2x_{t-1}^2 + 0.7\sigma_{t-1}^2$ and $\varepsilon_t \sim$ i.i.d. $N(0,1)$.[3] The series $\{x_t\}$ is not autocorrelated but is not i.i.d. This is one of the models examined in the Monte Carlo study of Dalla *et al.* (2019). They find that the standard testing procedures are a bit oversized (e.g. by around 8% when $k, m = 1$), while the robust tests are correctly sized. We choose a realization where this is evident. The simulated data for $\{x_t\}$ are given in the workfile "simdata_uni.wf1".

[2] Robust CB for zero autocorrelation provides a robust acceptance region for H_0.
[3] We initialize $\sigma_1^2 = \text{var}(x_t) = 10$, simulate 400 observations and drop the first 100.

On the main dialog box of the add-in, we select **Testing zero autocorrelation**. We specify the **Series name** x and set to 10 the **Lags to include**. By default, the value of the **Significance level** is $\alpha = 5\%$ and the value of the **Threshold** is $\lambda = 2.576$.



The graphs and the table with the results are provided in a spool called "x_actest".[4] We have the following testing outputs:



Tests for zero autocorrelation of X

Lag	AC	Stand. CB(95%)	Robust CB(95%)	Lag	t	p-value	t-tilde	p-value	Lag	LB	p-value	Q-tilde	p-value
1	0.169	(- 0.113, 0.113)	(- 0.257, 0.257)	1	2.929	0.003	1.292	0.196	1	8.664	0.003	1.669	0.196
2	0.157	(- 0.113, 0.113)	(- 0.238, 0.238)	2	2.726	0.006	1.296	0.195	2	16.194	0.000	3.348	0.187
3	-0.009	(- 0.113, 0.113)	(- 0.209, 0.209)	3	-0.153	0.878	-0.083	0.934	3	16.218	0.001	3.355	0.340
4	0.030	(- 0.113, 0.113)	(- 0.159, 0.159)	4	0.517	0.605	0.369	0.712	4	16.491	0.002	3.491	0.479
5	-0.054	(- 0.113, 0.113)	(- 0.155, 0.155)	5	-0.937	0.349	-0.682	0.495	5	17.390	0.004	3.957	0.556
6	-0.039	(- 0.113, 0.113)	(- 0.137, 0.137)	6	-0.678	0.498	-0.560	0.576	6	17.862	0.007	4.270	0.640
7	0.006	(- 0.113, 0.113)	(- 0.146, 0.146)	7	0.101	0.920	0.078	0.938	7	17.872	0.013	4.276	0.747
8	-0.045	(- 0.113, 0.113)	(- 0.132, 0.132)	8	-0.777	0.437	-0.664	0.507	8	18.497	0.018	4.717	0.787
9	-0.045	(- 0.113, 0.113)	(- 0.136, 0.136)	9	-0.775	0.438	-0.645	0.519	9	19.121	0.024	5.132	0.823
10	0.002	(- 0.113, 0.113)	(- 0.145, 0.145)	10	0.036	0.972	0.028	0.978	10	19.122	0.039	5.133	0.882

The left-hand side graph is plotting for maximum 10 lags, the sample autocorrelation $\widehat{\rho}_k$ ("AC"), the standard and robust CB(95%). The right-hand side graph is plotting for maximum 10 lags, the cumulative test statistics LB_m, $\widetilde{Q}_m$ and their critical values at 5% significance level ("cv(5%)"). The table reports the results of the graphs along with the p-values for all the statistics: standard t_k ("t") and LB_m ("LB") and robust $\widetilde{t}_k$ ("t-tilde") and $\widetilde{Q}_m$ ("Q-tilde").

[4]The name of the spool starts with the name of the series followed by "_actest".

From the left-hand side graph we can conclude that $H_0 : \rho_k = 0$ is rejected at $\alpha = 5\%$ when $k = 1, 2$ and is not rejected at $\alpha = 5\%$ when $k = 3, ..., 10$ using standard methods, but is not rejected at $\alpha = 5\%$ for any k using robust methods. From the right-hand side graph we can conclude that the cumulative hypothesis $H_0 : \rho_1 = ... = \rho_m = 0$ is rejected at $\alpha = 5\%$ for all m using standard methods, but is not rejected at any m using robust methods. Subsequently, from the p-values in the table we find that using standard methods, $H_0 : \rho_k = 0$ is rejected at $\alpha = 1\%$ when $k = 1, 2$ and is not rejected at $\alpha = 10\%$ when $k = 3, ..., 10$, whereas using robust methods it is not rejected at $\alpha = 10\%$ for any k. Using standard methods the cumulative hypothesis $H_0 : \rho_1 = ... = \rho_m = 0$ is rejected at $\alpha = 0.1\%$ for $m = 2$, at $\alpha = 1\%$ when $m = 1, 3, ..., 6$ and at $\alpha = 5\%$ for $m = 7, ..., 10$, whereas using robust methods it is not rejected at $\alpha = 10\%$ for any m. Overall, standard testing procedures show evidence of autocorrelation, although the series is not autocorrelated. The robust testing procedures provide the correct inference.

3 Testing zero cross-correlation

For a bivariate time series $\{x_t, y_t\}$, given a sample $(x_1, ..., x_n), (y_1, ..., y_n)$, the null hypothesis $H_0 : \rho_{xy,k} = 0$ of no cross-correlation at lag $k = 0, 1, 2, ...$ is tested at α significance level using the sample cross-correlation $\widehat{\rho}_{xy,k}$ and the $100(1-\alpha)\%$ confidence band (CB) for zero cross-correlation, obtained using the corresponding t-type statistics ($t_{xy,k}$ "standard" and $\widetilde{t}_{xy,k}$ "robust").[5] The null hypothesis $H_0 : \rho_{xy,0} = ... = \rho_{xy,m} = 0$ of no cross-correlation at cumulative lags $m = 0, 1, 2, ...$ is tested using portmanteau type statistics (Haugh-Box $HB_{xy,m}$ "standard" and $\widetilde{Q}_{xy,m}$ "robust"). The following notation is used.

Standard procedures:

$$CB(100(1-\alpha)\%) = (-z_{\alpha/2}/\sqrt{n}, z_{\alpha/2}/\sqrt{n}), \quad t_{xy,k} = \sqrt{n}\widehat{\rho}_{xy,k}, \quad HB_{xy,m} = n^2 \sum_{k=0}^{m} \frac{\widehat{\rho}^2_{xy,k}}{n-k}.$$

Robust procedures:

$$CB(100(1-\alpha)\%) = (-z_{\alpha/2}\frac{\widehat{\rho}_{xy,k}}{\widetilde{t}_{xy,k}}, z_{\alpha/2}\frac{\widehat{\rho}_{xy,k}}{\widetilde{t}_{xy,k}}), \quad \widetilde{t}_{xy,k} = \frac{\sum_{t=k+1}^{n} e_{xy,tk}}{\left(\sum_{t=k+1}^{n} e^2_{xy,tk}\right)^{1/2}}, \quad \widetilde{Q}_{xy,m} = \widetilde{t}'_{xy}\, \widehat{R}^{*\,-1}_{xy}\, \widetilde{t}_{xy},$$

where $e_{xy,tk} = (x_t - \bar{x})(y_{t-k} - \bar{y})$, $\bar{x} = n^{-1}\sum_{t=1}^{n} x_t$, $\bar{y} = n^{-1}\sum_{t=1}^{n} y_t$, $\widetilde{t}_{xy} = (\widetilde{t}_{xy,0}, ..., \widetilde{t}_{xy,m})'$ and $\widehat{R}^*_{xy} = (\widehat{r}^*_{xy,jk})$ is a matrix with elements $\widehat{r}^*_{xy,jk} = \widehat{r}_{xy,jk} I(|\tau_{xy,jk}| > \lambda)$ where λ is the threshold,

$$\widehat{r}_{xy,jk} = \frac{\sum_{t=\max(j,k)+1}^{n} e_{xy,tj} e_{xy,tk}}{(\sum_{t=\max(j,k)+1}^{n} e^2_{xy,tj})^{1/2} (\sum_{t=\max(j,k)+1}^{n} e^2_{xy,tk})^{1/2}}, \quad \tau_{xy,jk} = \frac{\sum_{t=\max(j,k)+1}^{n} e_{xy,tj} e_{xy,tk}}{(\sum_{t=\max(j,k)+1}^{n} e^2_{xy,tj} e^2_{xy,tk})^{1/2}}.$$

[5] Robust CB for zero cross-correlation provides a robust acceptance region for H_0.

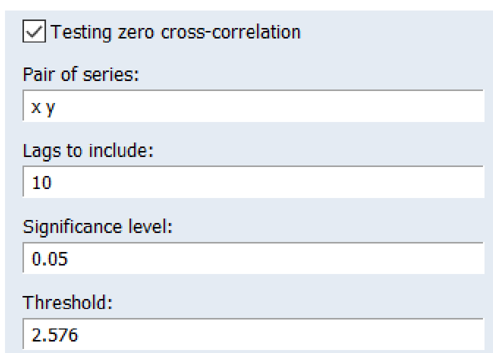
Applying standard and robust tests, at significance level α, $H_0 : \rho_{xy,k} = 0$ is rejected when $\widehat{\rho}_{xy,k} \notin CB(100(1-\alpha)\%)$ or $|t_{xy,k}|, |\widetilde{t}_{xy,k}| > z_{\alpha/2}$. In turn, $H_0 : \rho_{xy,0} = ... = \rho_{xy,m} = 0$ is rejected when $HB_{xy,m}, \widetilde{Q}_{xy,m} > \chi^2_{m,\alpha}$. Here, $z_{\alpha/2}$ and $\chi^2_{m,\alpha}$ stand for the upper $\alpha/2$ and α quantiles of $N(0,1)$ and χ^2_m distributions.

The above procedures where outlined for $k, m \geq 0$. For $k, m < 0$, the tests are analogously defined, noting that $\widehat{\rho}_{xy,k} = \widehat{\rho}_{yx,-k}$, $t_{xy,k} = t_{yx,-k}$, $\widetilde{t}_{xy,k} = \widetilde{t}_{yx,-k}$, $HB_{xy,m} = HB_{yx,-m}$, $\widetilde{Q}_{xy,m} = \widetilde{Q}_{yx,-m}$.

Example

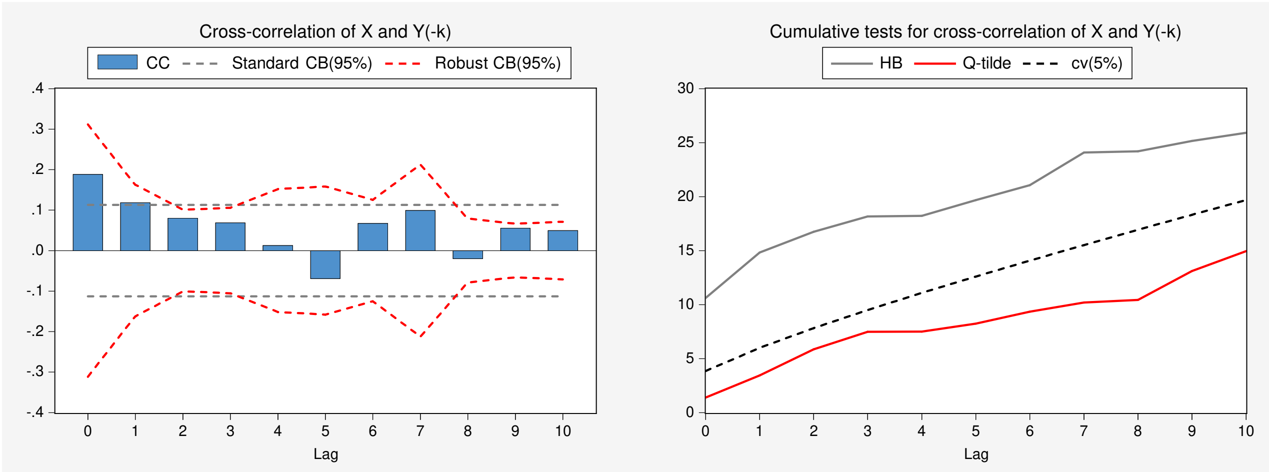
We provide an example to illustrate testing for zero cross-correlation of a bivariate time series $\{x_t, y_t\}$. We simulate $n = 300$ data as noise and SV-AR(1) using the same noise in the AR(1) part: $x_t = \varepsilon_t$ and $y_t = \exp(z_t)u_t$ with $z_t = 0.7z_{t-1} + \varepsilon_t$, $\varepsilon_t, u_t \sim$ i.i.d. $N(0,1)$, $\{\varepsilon_t\}$ and $\{u_t\}$ mutually independent.[6] The series $\{x_t\}$ and $\{y_t\}$ are uncorrelated but are not independent of each other, both are serially uncorrelated and only $\{x_t\}$ is i.i.d. This is one of the models examined in the Monte Carlo study of Dalla *et al.* (2019). They find that the standard testing procedures are rather oversized (e.g. by around 25% when $k, m = 0$), while the robust tests are correctly sized. We choose a realization where this is evident. The simulated data for $\{x_t, y_t\}$ are given in the workfile "simdata_biv_multi.wf1".

On the main dialog box of the add-in, we select **Testing zero cross-correlation**. We specify the **Pair of series** x y and set to 10 the **Lags to include**. By default, the value of the **Significance level** is $\alpha = 5\%$ and the value of the **Threshold** is $\lambda = 2.576$.



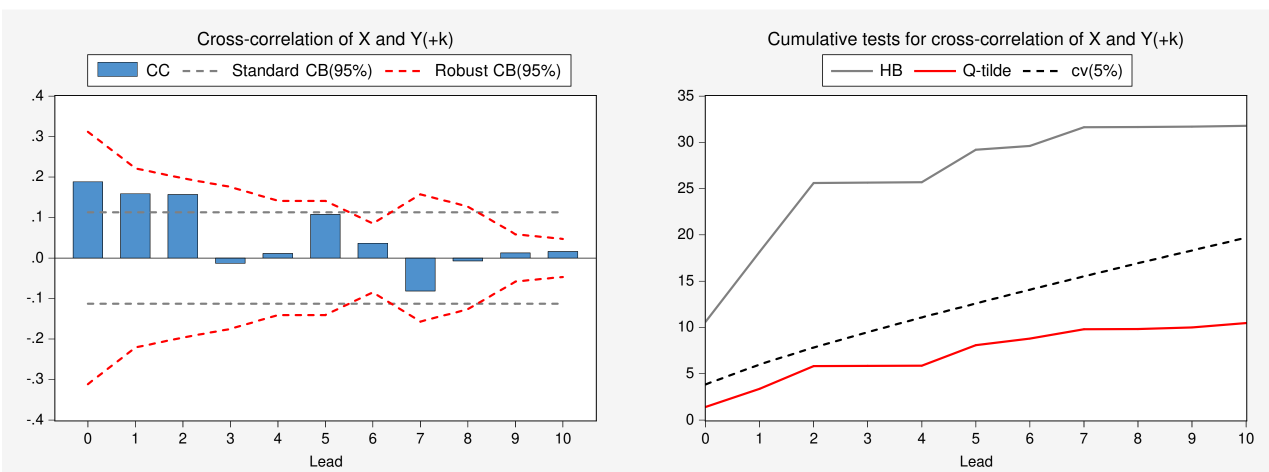
The graphs and the tables with the results are provided in a spool called "x_y_cctest".[7] We have the following testing outputs:

[6]We initialize $z_1 = Ez_t = 0$, simulate 400 observations and drop the first 100.
[7]The name of the spool starts with the names of the series followed by "_cctest".



Tests for cross-correlation of X and Y(-k)

Lag	CC	Stand. CB(95%)	Robust CB(95%)	Lag	t	p-value	t-tilde	p-value	Lag	HB	p-value	Q-tilde	p-value
0	0.188	(- 0.113, 0.113)	(- 0.312, 0.312)	0	3.259	0.001	1.183	0.237	0	10.621	0.001	1.400	0.237
1	0.118	(- 0.113, 0.113)	(- 0.162, 0.162)	1	2.046	0.041	1.426	0.154	1	14.822	0.001	3.434	0.180
2	0.080	(- 0.113, 0.113)	(- 0.100, 0.100)	2	1.384	0.166	1.560	0.119	2	16.750	0.001	5.867	0.118
3	0.068	(- 0.113, 0.113)	(- 0.106, 0.106)	3	1.186	0.236	1.269	0.204	3	18.170	0.001	7.477	0.113
4	0.012	(- 0.113, 0.113)	(- 0.152, 0.152)	4	0.215	0.830	0.160	0.873	4	18.217	0.003	7.503	0.186
5	-0.069	(- 0.113, 0.113)	(- 0.158, 0.158)	5	-1.197	0.232	-0.857	0.391	5	19.673	0.003	8.238	0.221
6	0.067	(- 0.113, 0.113)	(- 0.125, 0.125)	6	1.167	0.243	1.056	0.291	6	21.062	0.004	9.353	0.228
7	0.099	(- 0.113, 0.113)	(- 0.213, 0.213)	7	1.718	0.086	0.914	0.361	7	24.084	0.002	10.188	0.252
8	-0.020	(- 0.113, 0.113)	(- 0.079, 0.079)	8	-0.343	0.732	-0.490	0.624	8	24.205	0.004	10.428	0.317
9	0.055	(- 0.113, 0.113)	(- 0.066, 0.066)	9	0.959	0.337	1.637	0.102	9	25.154	0.005	13.109	0.218
10	0.049	(- 0.113, 0.113)	(- 0.071, 0.071)	10	0.855	0.392	1.360	0.174	10	25.911	0.007	14.958	0.184



Tests for cross-correlation of X and Y(+k)

Lead	CC	Stand. CB(95%)	Robust CB(95%)	Lead	t	p-value	t-tilde	p-value	Lead	HB	p-value	Q-tilde	p-value
0	0.188	(- 0.113, 0.113)	(- 0.312, 0.312)	0	3.259	0.001	1.183	0.237	0	10.621	0.001	1.400	0.237
1	0.159	(- 0.113, 0.113)	(- 0.221, 0.221)	1	2.746	0.006	1.405	0.160	1	18.185	0.000	3.375	0.185
2	0.157	(- 0.113, 0.113)	(- 0.197, 0.197)	2	2.713	0.007	1.562	0.118	2	25.597	0.000	5.815	0.121
3	-0.013	(- 0.113, 0.113)	(- 0.175, 0.175)	3	-0.229	0.819	-0.147	0.883	3	25.650	0.000	5.837	0.212
4	0.011	(- 0.113, 0.113)	(- 0.141, 0.141)	4	0.195	0.845	0.157	0.876	4	25.689	0.000	5.862	0.320
5	0.107	(- 0.113, 0.113)	(- 0.141, 0.141)	5	1.859	0.063	1.491	0.136	5	29.201	0.000	8.083	0.232
6	0.036	(- 0.113, 0.113)	(- 0.085, 0.085)	6	0.630	0.529	0.839	0.401	6	29.606	0.000	8.788	0.268
7	-0.081	(- 0.113, 0.113)	(- 0.157, 0.157)	7	-1.407	0.159	-1.013	0.311	7	31.634	0.000	9.813	0.278
8	-0.007	(- 0.113, 0.113)	(- 0.127, 0.127)	8	-0.122	0.903	-0.109	0.914	8	31.649	0.000	9.825	0.365
9	0.013	(- 0.113, 0.113)	(- 0.058, 0.058)	9	0.218	0.827	0.422	0.673	9	31.698	0.000	10.003	0.440
10	0.016	(- 0.113, 0.113)	(- 0.047, 0.047)	10	0.281	0.779	0.677	0.498	10	31.780	0.001	10.462	0.489

The left-hand side graphs are plotting for maximum ± 10 lags, the sample cross-correlation $\widehat{\rho}_{xy,k}$ ("CC"), the standard and robust CB(95%). The right-hand side graphs are plotting for maximum ± 10 lags, the cumulative test statistics $HB_{xy,m}$, $\widetilde{Q}_{xy,m}$ and their critical values at 5% significance level ("cv(5%)"). The tables report the results of the graphs along with the p-values for all the statistics: standard $t_{xy,k}$ ("t") and $HB_{xy,m}$ ("HB") and robust $\widetilde{t}_{xy,k}$ ("t-tilde") and $\widetilde{Q}_{xy,m}$ ("Q-tilde").

From the left-hand side graphs we can conclude that $H_0 : \rho_{xy,k} = 0$ is rejected at $\alpha = 5\%$ when $k = -2, -1, 0, 1$ and is not rejected at $\alpha = 5\%$ for $k \neq -2, -1, 0, 1$ using standard methods, but is not rejected at $\alpha = 5\%$ for any k using robust methods. From the right-hand side graphs we can conclude that the cumulative hypothesis $H_0 : \rho_{xy,0} = ... = \rho_{xy,m} = 0$ is rejected at $\alpha = 5\%$ for all m using standard methods, but is not rejected at any m using robust methods. Subsequently, from the p-values in the tables we find that using standard methods, $H_0 : \rho_{xy,k} = 0$ is rejected at $\alpha = 1\%$ when $k = -2, -1, 0$, at $\alpha = 5\%$ for $k = 1$, at $\alpha = 10\%$ when $k = -5, 7$ and is not rejected at $\alpha = 10\%$ for all $k \neq -5, -2, -1, 0, 1, 7$, whereas using robust methods it is not rejected at $\alpha = 10\%$ for any k. Using standard methods the cumulative hypothesis $H_0 : \rho_{xy,0} = ... = \rho_{xy,m} = 0$ is rejected at $\alpha = 0.1\%$ when $m = -10, ..., -1, 1, 2$ and at $\alpha = 1\%$ for $m = 0, 3, ..., 10$, whereas using robust methods it is not rejected at $\alpha = 10\%$ for any m. Overall, standard testing procedures show evidence of cross-correlation, although the series are uncorrelated from each other. The robust testing procedures provide the correct inference.

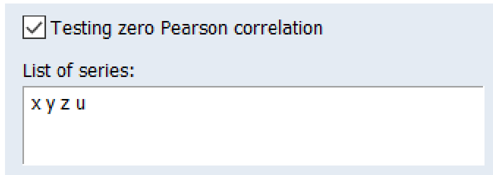
4 Testing zero Pearson correlation

For multivariate series $\{x_{1t}, ..., x_{pt}\}$, given a sample $(x_{11}, ..., x_{1n}), ..., (x_{p1}, ..., x_{pn})$, the null hypothesis $H_0 : \rho_{x_i x_j} = 0$ of no correlation between variables $\{x_{it}, x_{jt}\}$ is tested at α significance level using the sample Pearson correlation $\widehat{\rho}_{x_i x_j}$ and the p-value of the robust t-type statistic $\widetilde{t}_{x_i x_j}$. This robust procedure is obtained from the $\widetilde{t}_{xy,k}$ test of Section 3 setting $x = x_i$, $y = x_j$ and $k = 0$.

Example

We provide an example to illustrate testing zero correlation between variables of a 4-dimensional series $\{x_{1t}, x_{2t}, x_{3t}, x_{4t}\}$. We use the simulated data from the series $\{x_t, y_t, z_t, u_t\}$ of Section 3. The pairs $\{x_t, u_t\}$ and $\{z_t, u_t\}$ are independent, $\{x_t, y_t\}$ and $\{y_t, z_t\}$ are uncorrelated but are dependent, while $\{x_t, z_t\}$ and $\{y_t, u_t\}$ are correlated. From the four series only $\{x_t\}$ and $\{u_t\}$ are i.i.d. The simulated data for $\{x_t, y_t, z_t, u_t\}$ are given in the workfile "simdata_biv_multi.wf1".

On the main dialog box of the add-in, we select **Testing zero Pearson correlation**. We specify the **List of series** $x\ y\ z\ u$.



The tables with the results are provided in a spool called "x_u_rcorrtest".[8] We have the following testing outputs:

Matrix of Pearson correlations

	X	Y	Z	U
X	1	0.188	0.716	0.005
Y	0.188	1	0.280	0.210
Z	0.716	0.280	1	0.020
U	0.005	0.210	0.020	1

Matrix of p-values

	X	Y	Z	U
X		0.237	0.000	0.933
Y	0.237		0.169	0.003
Z	0.000	0.169		0.703
U	0.933	0.003	0.703	

The two tables report the sample Pearson correlations $\widehat{\rho}_{x_i x_j}$ among all pairs i, j of variables (left) and their p-values for testing significance of correlation (right).

From the p-values in the right-hand side table we can conclude that $H_0 : \rho_{xy} = 0$, $H_0 : \rho_{xu} = 0$, $H_0 : \rho_{yz} = 0$ and $H_0 : \rho_{zu} = 0$ are not rejected at $\alpha = 10\%$, $H_0 : \rho_{xz} = 0$ is rejected at $\alpha = 0.1\%$ and $H_0 : \rho_{yu} = 0$ is rejected at $\alpha = 1\%$. Overall, the robust testing procedure provides the correct inference. In contrast, the standard procedure[9] gives wrong inference when the series are uncorrelated but dependent. To demonstrate this, we use **Covariance Analysis** to evaluate the sample Pearson correlations and their p-values for testing significance of correlation.

We have the following outputs:

Covariance Analysis: Ordinary
Date: 29/01/20 Time: 09:00
Sample: 1 300
Included observations: 300

Correlation	X	Y	Z	U
X	1.000000			
Y	0.188161	1.000000		
Z	0.715752	0.280011	1.000000	
U	0.004634	0.209584	0.019612	1.000000

Probability	X	Y	Z	U
X	-----			
Y	0.0011	-----		
Z	0.0000	0.0000	-----	
U	0.9363	0.0003	0.7351	-----

[8] The name of the spool starts with the names of the first and the last series followed by "_rcorrtest".

[9] The standard procedure is implemented using **Covariance Analysis**. There, the standard t-test differs slightly from that given in Section 3. In **Covariance Analysis** the statistic $t'_{x_i x_j} = \widehat{\rho}_{x_i x_j} \sqrt{(n-2)/(1-\widehat{\rho}^2_{x_i x_j})}$ and critical values from the t_{n-2} distribution are used, while in Section 3 we take $t_{x_i x_j} = \sqrt{n}\, \widehat{\rho}_{x_i x_j}$ and critical values from the $N(0,1)$ distribution. For big samples, they give very similar results under H_0. For example, in Section 3 we find p-value of 0.00112 in testing $H_0 : \rho_{xy} = 0$ with the standard t_{xy} test, while in the output from **Covariance Analysis** it is 0.00106 using the standard t'_{xy} test.

From the p-values in the right-hand side table we can conclude that $H_0 : \rho_{xu} = 0$ and $H_0 : \rho_{zu} = 0$ are not rejected at $\alpha = 10\%$, $H_0 : \rho_{xz} = 0$, $H_0 : \rho_{yz} = 0$ and $H_0 : \rho_{yu} = 0$ are rejected at $\alpha = 0.1\%$ and $H_0 : \rho_{xy} = 0$ is rejected at $\alpha = 1\%$. Hence, using the standard procedure we wrongly conclude that the series $\{x_t\}$ with $\{y_t\}$ and $\{y_t\}$ with $\{z_t\}$ are correlated.

5 Testing i.i.d. property

For a univariate series $\{x_t\}$, given a sample $x_1, ..., x_n$, the null hypothesis of the i.i.d. property is tested at lag $k = 1, 2, ...$ by verifying $H_0 : \rho_{x,k} = 0, \rho_{|x|,k} = 0$ or $H_0 : \rho_{x,k} = 0, \rho_{x^2,k} = 0$, using the $J_{x,|x|,k}$ and $J_{x,x^2,k}$ statistics.[10] The null hypothesis of the i.i.d. property at cumulative lags $m = 1, 2, ...$ is tested by verifying $H_0 : \rho_{x,k} = 0, \rho_{|x|,k} = 0, k = 1, ..., m$ or $H_0 : \rho_{x,k} = 0, \rho_{x^2,k} = 0, k = 1, ..., m$, using the $C_{x,|x|,m}$ and $C_{x,x^2,m}$ statistics. The following notation is used.

$$J_{x,|x|,k} = \frac{n^2}{n-k}(\widehat{\rho}^2_{x,k} + \widehat{\rho}^2_{|x|,k}), \quad C_{x,|x|,m} = \sum_{k=1}^{m} J_{x,|x|,k},$$
$$J_{x,x^2,k} = \frac{n^2}{n-k}(\widehat{\rho}^2_{x,k} + \widehat{\rho}^2_{x^2,k}), \quad C_{x,x^2,m} = \sum_{k=1}^{m} J_{x,x^2,k},$$

where $\widehat{\rho}_{x,k} = \widehat{\text{corr}}(x_t, x_{t-k})$, $\widehat{\rho}_{|x|,k} = \widehat{\text{corr}}(|x_t - \bar{x}|, |x_{t-k} - \bar{x}|)$, $\widehat{\rho}_{x^2,k} = \widehat{\text{corr}}((x_t - \bar{x})^2, (x_{t-k} - \bar{x})^2)$ and $\bar{x} = n^{-1}\sum_{t=1}^{n} x_t$ with $\widehat{\text{corr}}$ denoting the sample correlation estimate.

Applying the tests, at significance level α, $H_0 : \rho_{x,k} = 0, \rho_{|x|,k} = 0$ or $H_0 : \rho_{x,k} = 0, \rho_{x^2,k} = 0$ is rejected when $J_{x,|x|,k} > \chi^2_{2,\alpha}$ or $J_{x,x^2,k} > \chi^2_{2,\alpha}$. In turn, $H_0 : \rho_{x,k} = 0, \rho_{|x|,k} = 0, k = 1, ..., m$ or $H_0 : \rho_{x,k} = 0, \rho_{x^2,k} = 0, k = 1, ..., m$ is rejected when $C_{x,|x|,m} > \chi^2_{2m,\alpha}$ or $C_{x,x^2,m} > \chi^2_{2m,\alpha}$. Here, $\chi^2_{m,\alpha}$ stands for the upper α quantile of χ^2_m distribution.

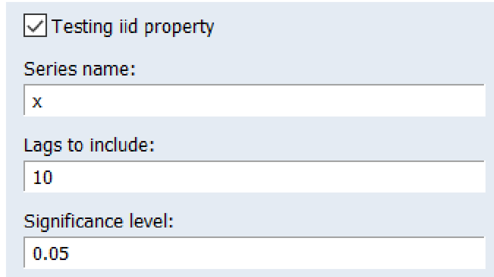
Example

We provide an example to illustrate testing for the i.i.d. property of a univariate series $\{x_t\}$. We use the simulated data from the series $\{x_t\}$ of Section 3. The series $\{x_t\}$ is i.i.d. The simulated data for $\{x_t\}$ are given in the workfile "simdata_biv_multi.wf1".

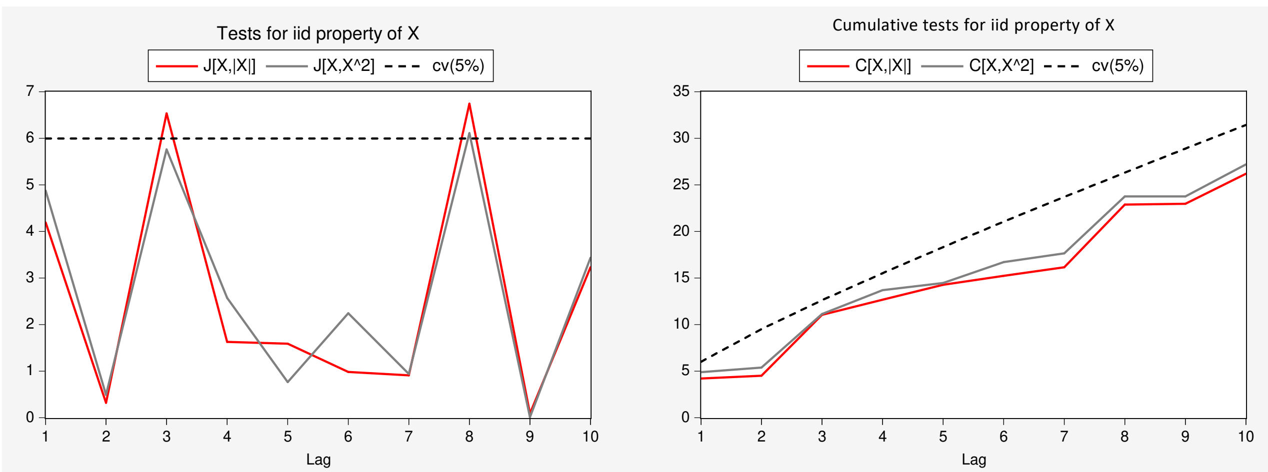
On the main dialog box of the add-in, we select **Testing iid property**. We specify the **Series name** x and set to 10 the **Lags to include**.[11] By default, the value of the **Significance level** is $\alpha = 5\%$.

[10] Notation: $\rho_{x,k} = \text{corr}(x_t, x_{t-k})$, $\rho_{|x|,k} = \text{corr}(|x_t - \mu|, |x_{t-k} - \mu|)$, $\rho_{x^2,k} = \text{corr}((x_t - \mu)^2, (x_{t-k} - \mu)^2)$ and $\mu = Ex_t$.

[11] The first letter of the series name is used as subscript in the statistics.



The graphs and the table with the results are provided in a spool "x_iidtest".[12] We have the following testing outputs:



Tests for iid property of X

Lag	J[X,|X|]	p-value	J[X,X^2]	p-value	Lag	C[X,|X|]	p-value	C[X,X^2]	p-value
1	4.189	0.123	4.876	0.087	1	4.189	0.123	4.876	0.087
2	0.317	0.853	0.489	0.783	2	4.507	0.342	5.365	0.252
3	6.534	0.038	5.757	0.056	3	11.041	0.087	11.122	0.085
4	1.626	0.444	2.571	0.277	4	12.666	0.124	13.692	0.090
5	1.586	0.452	0.763	0.683	5	14.252	0.162	14.455	0.153
6	0.979	0.613	2.243	0.326	6	15.231	0.229	16.698	0.161
7	0.906	0.636	0.940	0.625	7	16.138	0.305	17.638	0.224
8	6.741	0.034	6.110	0.047	8	22.878	0.117	23.748	0.095
9	0.090	0.956	0.012	0.994	9	22.968	0.192	23.759	0.163
10	3.228	0.199	3.436	0.179	10	26.196	0.159	27.195	0.130

The graphs are plotting for maximum 10 lags, the test statistics $J_{x,|x|,k}$, $J_{x,x^2,k}$ (left), the cumulative test statistics $C_{x,|x|,m}$, $C_{x,x^2,m}$ (right) and their critical values at 5% significance level ("cv(5%)"). The table reports the results of the graphs along with the p-values for all the statistics: $J_{x,|x|,k}$ ("J[x,|x|]"), $J_{x,x^2,k}$ ("J[x,x^2]"), $C_{x,|x|,m}$ ("C[x,|x|]") and $C_{x,x^2,m}$ ("C[x,x^2]").

From the left-hand side graph we can conclude that $H_0 : \rho_{x,k} = 0, \rho_{|x|,k} = 0$ is not rejected at $\alpha = 5\%$ for any k except $k = 3, 8$ or $H_0 : \rho_{x,k} = 0, \rho_{x^2,k} = 0$ is not rejected at $\alpha = 5\%$ for any k except $k = 8$. From the right-hand side graph we can conclude that the cumulative hypothesis $H_0 : \rho_{x,k} = 0, \rho_{|x|,k} = 0, k = 1, ..., m$ or $H_0 : \rho_{x,k} = 0, \rho_{x^2,k} = 0, k = 1, ..., m$ is not rejected at $\alpha = 5\%$ for any m. Subsequently, from the p-values in the table we find that

[12]The name of the spool starts with the name of the series followed by "_iidtest".

$H_0 : \rho_{x,k} = 0, \rho_{|x|,k} = 0$ is rejected at $\alpha = 5\%$ for $k = 3, 8$ and is not reject at $\alpha = 10\%$ when $k \neq 3, 8$ or $H_0 : \rho_{x,k} = 0, \rho_{x^2,k} = 0$ is rejected at $\alpha = 5\%$ for $k = 8$ and at $\alpha = 10\%$ for $k = 1, 3$ and is not rejected at $\alpha = 10\%$ when $k \neq 1, 3, 8$. The cumulative hypothesis $H_0 : \rho_{x,k} = 0, \rho_{|x|,k} = 0, k = 1, ..., m$ is rejected at $\alpha = 10\%$ for $m = 3$ and is not rejected at $\alpha = 10\%$ when $m \neq 3$ or $H_0 : \rho_{x,k} = 0, \rho_{x^2,k} = 0, k = 1, ..., m$ is rejected at $\alpha = 10\%$ for $m = 1, 3, 4, 8$ and is not rejected at $\alpha = 10\%$ for $m \neq 1, 3, 4, 8$. Overall, the testing procedures provide the correct inference.

6 Remarks

The theory and Monte Carlo study in Dalla *et al.* (2019) suggest that:

(i) In testing for autocorrelation the series needs to have constant mean.

(ii) In testing for cross-correlation each of the series needs to have constant mean and to be serially uncorrelated when applying the portmanteau type statistics or at least one when applying the t-type tests.

(iii) In testing for Pearson correlation at least one of the series needs to have constant mean and to be serially uncorrelated.

(iv) For relatively large lag it may happen that the robust portmanteau statistic is negative. In such a case, missing values ("NA") are reported for the statistic and its p-value.

(v) The values $\lambda = 1.96, 2.576$ are good candidates for the threshold in the robust portmanteau statistics, with $\lambda = 2.576$ performing better at relatively large lags.

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